Introduction

This is a chronology of my journey through a sixty-year obsession with physics. I will focus on the things that have surprised me, things so obvious that I wondered why I hadn't learned them in school? Take the equation for the period of the pendulum: $T = 2\pi\sqrt{\frac{L}{g}}$, where L is the length of the pendulum, and g is the acceleration of gravity. By substituting R (radius of a circular orbit) for L, and $\frac{GM}{R^2}$ (gravitational acceleration toward a mass, M) for g, the equation for the pendulum becomes a trivial algebraic permutation of Kepler's third law:

$$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{R^3}{GM}}, \text{ or } \quad \frac{GM}{4\pi^2} = \frac{R^3}{T^2}$$

This paper is for anyone with a keen interest in physics, but doesn't want to spend sixty years figuring out simple stuff that can't be found in physics texts.

The Fine Structure Constant, $\alpha \approx \frac{1}{137}$

Alpha is the ratio of hydrogen's ground state circular orbital velocity, to light speed.

Arnold Sommerfeld found α by applying the following principle from orbital mechanics: **The escape energy of a circular orbit is twice the kinetic energy**. Rydberg had found the ionization energy required to completely disassociate the hydrogen ground state electron: $R_Y = 13.6eV$.

Since half escape energy equals kinetic energy, the other half must be Rydberg energy.

$\therefore KE = \frac{1}{2}m_e v^2 = R_Y = 13.6eV.$

$\therefore E_{esc} = 2R_Y = m_e v^2 = 27.2\ eV$, where $m_e c^2 = 511{,}000\ eV$

$$\therefore\ \frac{1}{\alpha} = \frac{c}{v} = \sqrt{\frac{m_e c^2}{m_e v^2}} = \sqrt{\frac{511{,}000eV}{27.2eV}} \approx 137$$

(For the published CODATA value of $\frac{1}{\alpha}$, use CODATA values for R_Y and $m_e c^2$.)

Compton wavelength, λ_e, the Classical Electron Radius, r_e, and 861

The electron's Compton wavelength, $\lambda_e = 2.426e - 12\ m$, and the classical electron radius, $r_e = 2.818e - 15\ m$, have similar definitions:
λ_e is the wavelength of light with photon energy equal to $m_e c^2$.
r_e is the distance between **two** quantum charges where their relative electrical potential energy is $\pm m_e c^2$. (Escape energy required to separate two opposite sign charges to infinity, or energy required to bring two like sign charges from infinity to r_e.)

The ratio $\dfrac{\lambda_e}{r_e}$ is approximately 860.894, remarkably close to the integer 861.

The number 861 is interesting, because it's a Triangular number!
(A set of **42 unique items** will yield exactly **861 unique pairs**.) This property of Triangular numbers might have significant implications for a collection of entities which all interact with each other. (Like quantum charges.) Another unexpected property of 861 emerges in the next section.

Geometry of an 861-sided regular polygon

For a regular polygon with N sides, the central angle subtended by one side is $\theta = \dfrac{2\pi}{N}$. If N=861 then $\theta = \dfrac{2\pi}{861}$.

Numerically, $\dfrac{2\pi}{861} \approx 0.007298$.

The fine-structure constant is $\alpha \approx \dfrac{1}{137} \approx 0.007299.$ $\quad \therefore \alpha \approx \dfrac{2\pi}{861}$.

For a polygon with unit sides, the radius of the circumscribed circle is
$R = \dfrac{1}{2\sin(\pi/N)}$.

With N = 861, $R = 137$: $\quad \dfrac{861}{137} \div 2\pi \approx 1.00024. \qquad \therefore\ 2\pi \approx \dfrac{861}{137} \approx 861\ \alpha$

Thus, 861 links Compton wavelength, Fine Structure Constant, and the Classical Electron Radius.

Circular Motion in the Bohr Model

In the Bohr model of hydrogen, the electron in the ground state moves in a circular orbit with velocity $v = \alpha c$.

Thus the fine-structure constant determines the ratio between the electron's orbital velocity and the speed of light.

The Bohr quantization condition is: $m_e v a_0 = \hbar$.

If the circulation of the orbit is defined as $\Gamma = v(2\pi a_0)$, then

multiplying by the electron mass gives: $m_e \Gamma = m_e v(2\pi a_0)$.

Substituting the Bohr condition gives: $\hbar = m_e \dfrac{\Gamma}{2\pi}$.

This allows Planck's constant to be interpreted as **mass multiplied by circulation** for the electron orbit.

Centrifugal Force on the Electron in the Bohr Model

The inward (centripetal) acceleration on a body in a circular trajectory is well known as the trajectory velocity squared, divided by the radius of the circular trajectory. The outward (centrifugal) reaction force is merely the mass of the electron, times the centripetal acceleration:

$$F_{out} = \frac{(\alpha c)^2}{a_0} m_e = \frac{c^2}{137^4 r_e} m_e = \frac{r_e}{r_e} \frac{c^2}{137^4 r_e} m_e = \frac{r_e c^2}{a_0^2} m_e = \frac{r_e c^2}{r^2} m_e = F_{Coulomb}$$

The Coulomb force of attraction between the proton nucleus, and the orbiting electron must be equal to centrifugal force on the electron. Therefore, the last expression of the above equation is equivalent to the Coulomb constant, giving the force between two quantum charges, separated by r.

$$F_{Coulomb} = \pm \frac{Z r_e c^2}{r^2} \times Y m_e \quad \sim m/s^2 \times \; kg \quad \sim N$$

(The product YZ is the total unique pairs of Y - Z quantum charges.)

$\dfrac{Z r_e c^2}{r^2}$ might be interpreted as the spherical acceleration field around Z charges, while radial force is mass, $Y m_e$ times acceleration.

Electric Force between protons

The proton is 1836 times more massive than the electron, but the force between two protons, is exactly the same as the force between two electrons separated by the same distance: $F_{Coulomb} = \frac{r_e c^2}{r^2} m_e = \frac{r_e c^2}{r^2} \left(\frac{1}{1836}\right)^4 m_p$

The electrical acceleration field (times r^2) around either proton is:

$$\ddot{r}\ r^2 = r_e c^2 \left(\frac{1}{1836}\right)^4 = (2.818e-15)9e16/1836^4 = 2.232e-11 \sim m^3/s^2$$

$\left(\frac{1}{1836}\right)^2$ is the product of two (electron mass/proton mass) ratios, indicating the small fraction of charge pairs in a proton pair. Another $\left(\frac{1}{1836}\right)^2$ is the net reduction of both acceleration fields so that each proton, embedded in the other's acceleration field results in the same force as an electron embedded in an electron field.

A Numerical curiosity:

Note that: $3 \times 2.232e-11 = 6.696e-11 \sim m^3/s^2 \approx G$

Multiplying 3 times the above equation is approximately equal to the numerical value of the gravitational constant, but without the expected "kg" in the denominator of the dimensions, I wondered if this was just a numerical coincidence.

1. I was calculating the electric field of a proton (a nucleon), and it turned out to be numerically one-third weaker than the gravitational field associated with any nucleon, either a proton or a neutron.

2. One similarity between protons and neutrons is that they both have **three** quarks.

3. Given nature's propensity for reusing fundamental algorithms, I wondered whether G might be the smallest template for Kepler's third law. (2/3 e-10) ~ $m^3/kg\ s^2$

How do we measure the mass of the Sun?

We start with knowing the value of the gravitational constant, and if we assume $G = 6.666...e - 11 \sim m^3/kg\ s^2$, then, knowing the gravitational parameter from observing planetary data, $GM = 1.327e20\ \sim\ m^3/s^2$:

$$M = \frac{GM}{G} = \frac{1.327e20}{6.666e - 11} = 1.99e30 \approx 2e30\ \ kg$$

(I was starting to feel I was being punked! Where was the hidden camera?)

Constants that are integer multiples of the classical electron radius:

Compton Wavelength $\lambda_e = 861\ r_e$
Bohr radius: $a_0 = 137^2\ r_e$
Bohr Circumference: $C = 861 \times 137\ r_e$

Circular Motion in Planetary Orbits

Analysis of several approximately two-body solar orbits suggests approximate integer scaling of orbital parameters that is consistent with Kepler's third law:

$$GM = 4\pi^2 \frac{R_N^3}{T_N^2}$$

Where: $R_N^3 = (R_{\min} N^2)^3 = R_{min}^3 N^6$

And: $T_N^2 = (T_{\min} N^3)^2 = T_{min}^2 N^6$

$$\therefore\ \frac{R_N^3}{T_N^2} = \frac{R_{min}^3}{T_{min}^2}$$

Let V_Γ equal the maximum orbit velocity at R_{min} and T_{min}, where:

$$V_\Gamma = \frac{2\pi R_{\min}}{T_{\min}}\ .$$

Gravitational acceleration becomes: $g_N = \dfrac{g_{\max}}{N^4}$.

Thus four simple scaling relations appear:

$$V_N = \frac{V_\Gamma}{N}$$

$$R_N = R_{\min} N^2$$

$$T_N = T_{\min} N^3$$

$$g_N = \frac{g_{\max}}{N^4}.$$

Empirical Velocity Scale

Using orbital data for several Solar System bodies that most closely approximate two-body motion gives an empirical estimate of $V_\Gamma \approx 144$ km/s . Bodies used in the estimate include:

Body	Approximate N
Mercury	3
Mars	6
Ceres	8
Jupiter	11
Saturn	15

Venus and Earth were excluded because they form a strongly coupled pair exhibiting an approximate **8-year resonance.**

Comparison with Atomic Constants

Dividing the velocity scale by the speed of light gives: $\frac{V_\Gamma}{c} \approx 4.8 \times 10^{-4}$.

The square of three times the fine-structure constant is: $(3\alpha)^2 \approx 4.79 \times 10^{-4}$.

Thus $V_\Gamma \approx (3\alpha)^2 c$ to within the scatter of the planetary data.

Whether this numerical agreement reflects a deeper physical connection or a coincidence is left as an open question.

How was the equation for V_Γ found, and why should anyone care about V_Γ?

First, why should we care?

Atomic physics and planetary dynamics are normally treated as separate domains governed by vastly different length and energy scales. The appearance of the same dimensionless combination in both contexts motivated the search for additional scaling relationships."

Second, how was V_Γ found?

The V_Γ/N sequence, and the approximate V_Γ value, were first identified in a paper written by this author in 1975. The equation for V_Γ was only discovered after noticing that 3α was an integer fraction of 2π because 3 is a factor of 861: $3\alpha = \dfrac{2\pi}{287}$.

The choice of $(3\alpha)^2\ c$, was not based on a derivation, but on a numerical exploration motivated by the appearance of 861, 287, and α.

Final Observation

One notable feature of Kepler's law is that bodies of very different mass follow the same orbital scaling. For example, both the dwarf planet **Ceres** and the giant planet **Jupiter** follow the same Kepler scaling in their motion around the Sun. This is analogous to Galileo's observation that the period of a pendulum depends on its length and the gravitational acceleration, but not on the mass of the pendulum bob.

While I found the equivalence between Galileo's pendulum and Kepler's third law interesting, I had no clue that a real physics lesson would emerge from the comparison.

These examples suggest that the trajectories themselves are properties of the surrounding gravitational-inertial geometry. In this sense, the path followed by an orbiting body is determined primarily by the structure of the field rather than by the mass of the orbiting object.

Charles deAnne,

author of "THE MATTERHORN PARADOX" and "Insubordinate"